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Collinear improvement of the BFKL kernel in the electroproduction of two light vector mesons

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Abstract. The use of the BFKL kernel improved by the inclusion of subleading terms generated by renormalization group (RG) analysis has been suggested to cure the instabilities in the behavior of the BFKL Green's function in the next-to-leading approximation (NLA). We test the performance of a RG-improved kernel in the determination of the amplitude of a physical process, the electroproduction of two light vector mesons, in the BFKL approach in the NLA. We find that a smooth behavior of the amplitude with the center-of-mass energy can be achieved, setting the renormalization and energy scales appearing in the subleading terms to values much closer to the kinematical scales of the process than in the approaches based on the unimproved kernel.

1 Introduction

It is known that hard processes in which the center-of-mass energy is much larger than all the other scales are the natural ground for the application of the BFKL approach [1–4]. This approach was originally developed in the leading logarithmic approximation (LLA), which means resummation of all terms of the form $(\alpha_s \ln(s))^n$. In such an approximation the argument μ_R of the running coupling and the energy scale are not fixed. This motivated the extension of the approach to the next-to-leading logarithmic approximation (NLLA), which means resummation of all terms proportional to $\alpha_s(\alpha_s \ln(s))^n$. In both approximations the BFKL amplitude appears as a convolution of the Green's function of two interacting reggeized gluons with the impact factors of the colliding particles (see, for example, Fig. 1). The Green's function, which carries the dependence on the center-of-mass energy, can be determined through the BFKL equation. The impact factors are process-dependent and describe the interaction between reggeized gluons and scattering particles.

The singlet kernel of the BFKL equation in the next-to-leading approximation (NLA) was obtained for the forward case in [5, 6], completing the long program of calculation of the NLA corrections [7–20] (for a review, see [21]). In the non-forward case the ingredients for the NLA BFKL kernel have been known for a few years in the case of the color octet representation in the t-channel [22–26]. This color representation is very important to check the consistency of the s-channel unitarity with the gluon reggeization, i.e. for the "bootstrap" [27–36]. More recently, the last missing piece for the determination of the non-forward NLA BFKL kernel has been calculated in the singlet color representation, i.e. in the pomeron channel, relevant for physical applications [37–39]. The singlet NLA BFKL kernel in the so-called "dipole form" is available now also in the coordinate representation [40–42], which allows for the study of its conformal properties and the comparison with the kernel of the Balitsky–Kovchegov [43, 44] equation in the linear regime. So far, the color dipole kernel has been calculated in the NLA only for the quark part [45] and agrees with the dipole form of the quark part of the NLA BFKL kernel.

In this paper we will focus on the BFKL approach in the NLA and in the case of forward scattering. It is well known that the NLA corrections to the Green's function turn out to be large, this being a signal of the poor convergence of the BFKL series. In order to "cure" the resulting instability, more convergent kernels have been introduced, including terms generated by renormalization group (RG), or collinear, analysis [46]. They are based on the ω-shift method [46], with ω being the variable Mellin-conjugated to the squared center-of-mass energy s. The main effect of this method is that the scale-invariant part of the kernel eigenvalues carries a dependence on the Mellin variable ω, in such a way that the position of the singularities of the Green's function in the ω-plane becomes the solution of an implicit equation in ω. Many other studies have been performed, either based on this kind of improved kernels [47–55] or analyzing different aspects of the kernel NLA and alternative approaches [56–73]. The effects of these collinear corrections in exclusive observables have

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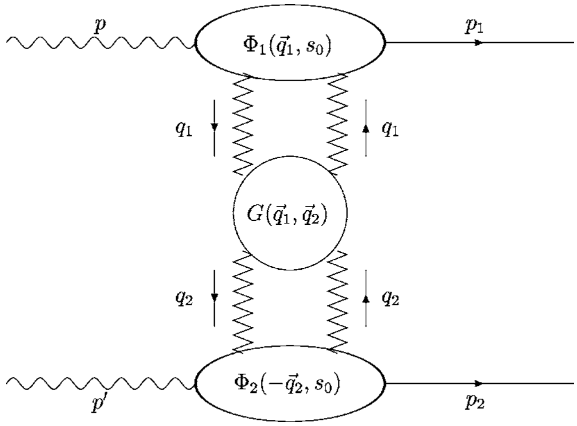

Fig. 1. Schematic representation of the amplitude for $\gamma^*(p)\,\gamma^*(p') \to V(p_1)\,V(p_2)$ forward scattering

been investigated in [74–76], with a posteriori confirmation in [77].

In [78] the original approach of [46] was revisited and an approximation to the original ω-shift was performed, leading to an explicit expression for the RG-improved NLA kernel. It was shown that this improved kernel leads to a NLA BFKL Green's function exempt of instabilities. Since the effect of the RG-improvement is to modify the BFKL kernel by the inclusion of terms beyond the NLA, one is led to conclude that RG-generated terms, although formally subleading, play an important numerical role in practical applications.

It is very interesting to test the RG-improvement of the kernel in the calculation of a full physical amplitude, rather than just considering its effect on the BFKL Green's function, and to compare it with other approaches. A test-field for this comparison can be provided by the physical process $\gamma^*\gamma^* \to VV$, where γ^* represents a virtual photon and V a light neutral vector meson (ρ^0, ω, ϕ). The amplitude of this reaction has been calculated in [79] through the convolution of the (unimproved) BFKL Green's function with the $\gamma^* \to V$ impact factors, calculated in [80, 81].[1] In the case of equal photon virtualities, the so-called "pure" BFKL regime, a numerical calculation has shown that NLA corrections are large and of opposite sign with respect to the leading order and are dominated, at the lower energies, by the NLA corrections from the impact factors. Nonetheless, an amplitude for this process with a smooth behavior in s could be achieved by "optimizing" the choice of the energy scale s_0 and of the renormalization scale $\mu_{\rm R}$, which appear in the subleading terms. Later on it has been found that the result is rather stable under change of the method of optimization of the perturbative series and of the representation adopted for the amplitude [82].

The striking feature of these investigations was that in all cases the optimal values of the two energy parameters turned out to be quite far from the kinematical scales of the reaction. For example, the optimal value of the renormalization scale $\mu_{\rm R}$ turned out to be typically as large as $\sim 10Q$, Q^2 being the virtuality of the colliding photons. The proposed explanation for these "unnatural" values was that they mimic the unknown next-to-NLA corrections, which should be large and of opposite sign with respect to the NLA in order to preserve the renormalization and energy scale invariance of the exact amplitude. If this explanation is correct and if the RG-improvement of the kernel catches the essential dynamics from subleading orders, then, by repeating the numerical determination of the $\gamma^*\gamma^* \to VV$ amplitude with the use of an RG-improved kernel, one should get more "natural" values for the optimal choices of the energy scales and, of course, results consistent with the previous determinations. In this work we address this question by calculating the NLA amplitude of the $\gamma^*\gamma^* \to VV$ process in the BFKL approach with the RG-improved kernel of [78], which can be straightforwardly implemented in the numerical set up of [79, 82].

The paper is organized as follows: in the next section we repeat the steps of [79, 82] to build up the NLA amplitude in two representations, series and "exponentiated", which implement the RG-improved kernel of [78]; in Sect. 3 we numerically evaluate the amplitude, considering both the cases of colliding photons with the same virtualities and with strongly ordered virtualities. We stress that in [79, 82] only the case of equal photons' virtualities was considered; attempts to determine the amplitude for strongly ordered virtualities were unsuccessful, due to the large instabilities met in the numerical analysis [83]. We expect that the RG-improvement should be even more effective in the latter case, since it was conceived to work in a kinematics with strong asymmetry in the transverse momentum plane [46].

2 The NLA amplitude with the RG-improved Green's function[1]

We consider the production of two light vector mesons ($V = \rho^0, \omega, \phi$) in the collision of two virtual photons,

$$\gamma^*(p)\gamma^*(p') \to V(p_1)V(p_2)\,. \quad (1)$$

Here, p_1 and p_2 are taken as Sudakov vectors satisfying $p_1^2 = p_2^2 = 0$ and $2(p_1 p_2) = s$; the virtual photon momenta are instead

$$p = \alpha p_1 - \frac{Q_1^2}{\alpha s} p_2\,, \quad p' = \alpha' p_2 - \frac{Q_2^2}{\alpha' s} p_1\,, \quad (2)$$

so that the photon virtualities become $p^2 = -Q_1^2$ and $(p')^2 = -Q_2^2$. We consider the kinematics when

$$s \gg Q_{1,2}^2 \gg \Lambda_{\rm QCD}^2\,, \quad (3)$$

[1] This amplitude has been considered also in [86–88].

[2] This section follows closely both Sect. 2 of [79] and Sect. 2 of [82], the only difference being the use of a modified BFKL kernel. The reader already familiar with the notation and the previous papers may prefer to go straight to the main formulas: (36) for the "exponentiated" representation, (37) for the "series" representation of the amplitude and (22) for the extra term in the BFKL kernel eigenvalue.

and

$$\alpha = 1 + \frac{Q_2^2}{s} + \mathcal{O}(s^{-2}), \quad \alpha' = 1 + \frac{Q_1^2}{s} + \mathcal{O}(s^{-2}). \quad (4)$$

In this case the vector mesons are produced by longitudinally polarized photons in the longitudinally polarized state [80, 81]. Other helicity amplitudes are power suppressed, with a suppression factor $\sim m_V/Q_{1,2}$. We will discuss here the amplitude of the forward scattering, i.e. when the transverse momenta of the produced V mesons are zero or when the variable $t = (p_1 - p)^2$ takes its maximal value, $t_0 = -Q_1^2 Q_2^2/s + \mathcal{O}(s^{-2})$.

The forward amplitude in the BFKL approach may be presented as follows

$$\mathrm{Im}_s(\mathcal{A}) = \frac{s}{(2\pi)^2} \int \frac{\mathrm{d}^2\mathbf{q}_1}{\mathbf{q}_1^2} \Phi_1(\mathbf{q}_1, s_0) \int \frac{\mathrm{d}^2\mathbf{q}_2}{\mathbf{q}_2^2} \Phi_2(-\mathbf{q}_2, s_0) \times \int_{\delta - \mathrm{i}\infty}^{\delta + \mathrm{i}\infty} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\mathbf{q}_1, \mathbf{q}_2). \quad (5)$$

This representation for the amplitude is valid with NLA accuracy.

In (5), $\Phi_1(\mathbf{q}_1, s_0)$ and $\Phi_2(-\mathbf{q}_2, s_0)$ are the impact factors describing the transitions $\gamma^*(p) \to V(p_1)$ and $\gamma^*(p') \to V(p_2)$, respectively. The Green's function in (5) is determined by the BFKL equation

$$\delta^2(\mathbf{q}_1 - \mathbf{q}_2) = \omega G_\omega(\mathbf{q}_1, \mathbf{q}_2) - \int \mathrm{d}^2\mathbf{q} K(\mathbf{q}_1, \mathbf{q}) G_\omega(\mathbf{q}, \mathbf{q}_2), \quad (6)$$

where $K(\mathbf{q}_1, \mathbf{q}_2)$ is the BFKL kernel. It is convenient to work in the transverse momentum representation, where "transverse" refers to the plane orthogonal to the vector mesons momenta. In this representation, defined by

$$\hat{\mathbf{q}}|\mathbf{q}_i\rangle = \mathbf{q}_i|\mathbf{q}_i\rangle, \quad (7)$$

$$\langle\mathbf{q}_1|\mathbf{q}_2\rangle = \delta^{(2)}(\mathbf{q}_1 - \mathbf{q}_2),$$

$$\langle A|B\rangle = \langle A|\mathbf{k}\rangle\langle\mathbf{k}|B\rangle = \int \mathrm{d}^2k A(\mathbf{k}) B(\mathbf{k}), \quad (8)$$

the kernel of the operator $\hat{K}$ is

$$K(\mathbf{q}_2, \mathbf{q}_1) = \langle\mathbf{q}_2|\hat{K}|\mathbf{q}_1\rangle, \quad (9)$$

and the equation for the Green's function reads

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega, \quad (10)$$

its solution being

$$\hat{G}_\omega = (\omega - \hat{K})^{-1}. \quad (11)$$

To clearly indicate the RG-improved pieces of the kernel, we decompose $\hat{K}$ as

$$\hat{K} = \bar{\alpha}_\mathrm{s}\hat{K}^0 + \bar{\alpha}_\mathrm{s}^2\hat{K}^1 + \hat{K}_\mathrm{RG}, \quad (12)$$

where

$$\bar{\alpha}_\mathrm{s} = \frac{\alpha_\mathrm{s} N_c}{\pi} \quad (13)$$

and N_c is the number of colors. In (12) $\hat{K}^0$ is the BFKL kernel in the LLA, $\hat{K}^1$ is the NLA correction and $\hat{K}_\mathrm{RG}$ includes the RG-generated terms, which are $\mathcal{O}(\bar{\alpha}_\mathrm{s}^3)$. The impact factors are also presented as an expansion in α_s

$$\Phi_{1,2}(\mathbf{q}) = \alpha_\mathrm{s} D_{1,2} \left[C_{1,2}^{(0)}(\mathbf{q}^2) + \bar{\alpha}_\mathrm{s} C_{1,2}^{(1)}(\mathbf{q}^2)\right],$$

$$D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1}, \quad (14)$$

where f_V is the meson dimensional coupling constant ($f_\rho \approx 200\,\mathrm{MeV}$) and e_q should be replaced by $e/\sqrt{2}$, $e/(3\sqrt{2})$ and $-e/3$ for the case of ρ^0, ω and ϕ meson production, respectively.

In the collinear factorization approach the meson transition impact factor is given as a convolution of the hard scattering amplitude for the production of a collinear quark–antiquark pair with the meson distribution amplitude (DA). The integration variable in this convolution is the fraction z of the meson momentum carried by the quark ($\bar{z} \equiv 1 - z$ is the momentum fraction carried by the antiquark):

$$C_{1,2}^{(0)}(\mathbf{q}^2) = \int_0^1 \mathrm{d}z \frac{\mathbf{q}^2}{\mathbf{q}^2 + z\bar{z}Q_{1,2}^2} \phi_\parallel(z). \quad (15)$$

The NLA correction to the hard scattering amplitude, for a photon with virtuality equal to Q^2, is defined as follows:

$$C^{(1)}(\mathbf{q}^2) = \frac{1}{4N_c} \int_0^1 \mathrm{d}z \frac{\mathbf{q}^2}{\mathbf{q}^2 + z\bar{z}Q^2} [\tau(z) + \tau(1-z)] \phi_\parallel(z), \quad (16)$$

with $\tau(z)$ given in (75) of [80]. $C_{1,2}^{(1)}(\mathbf{q}^2)$ are given by the previous expression with Q^2 replaced everywhere in the integrand by Q_1^2 and Q_2^2, respectively. We will use the DA in the asymptotic form $\phi_\parallel^\mathrm{as}(z) = 6z(1-z)$.

To determine the amplitude with NLA accuracy we need an approximate solution of (11). With the required accuracy this solution is

$$\hat{G}_\omega = (\omega - \bar{\alpha}_\mathrm{s}\hat{K}^0)^{-1} + (\omega - \bar{\alpha}_\mathrm{s}\hat{K}^0)^{-1} \left(\bar{\alpha}_\mathrm{s}^2\hat{K}^1 + \hat{K}_\mathrm{RG}\right) \times (\omega - \bar{\alpha}_\mathrm{s}\hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_\mathrm{s}^2\hat{K}^1\right)^2\right]. \quad (17)$$

Different from [79, 82], where $\hat{K}_\mathrm{RG}$ was absent, this Green's function includes effects that are beyond the NLA. The basis of eigenfunctions of the LLA kernel,

$$\hat{K}^0|\nu\rangle = \chi(\nu)|\nu\rangle,$$

$$\chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2} + \mathrm{i}\nu\right) - \psi\left(\frac{1}{2} - \mathrm{i}\nu\right), \quad (18)$$

is given by the following set of functions:

$$\langle \mathbf{q}|\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\mathbf{q}^2\right)^{\mathrm{i}\nu-\frac{1}{2}}\,, \tag{19}$$

for which the orthonormality condition takes the form

$$\langle \nu'|\nu\rangle = \int \frac{\mathrm{d}^2\mathbf{q}}{2\pi^2}\left(\mathbf{q}^2\right)^{\mathrm{i}\nu-\mathrm{i}\nu'-1} = \delta(\nu-\nu')\,. \tag{20}$$

The action of the modified BFKL kernel on these functions may be expressed as follows:

$$\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\chi(\nu)|\nu\rangle + \bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R})\\ &\times\left(\chi^{(1)}(\nu)+\frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle\\ &+\bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R})\frac{\beta_0}{4N_c}\chi(\nu)\left(\mathrm{i}\frac{\partial}{\partial\nu}\right)|\nu\rangle+\chi_\mathrm{RG}(\nu)|\nu\rangle\,,\end{aligned} \tag{21}$$

where the first term represents the action of LLA kernel, the second and the third ones stand for the diagonal and the non-diagonal parts of the NLA BFKL kernel [79] and

$$\begin{aligned}\chi_\mathrm{RG}(\nu) &= 2\\ &\times\mathrm{Re}\left\{\sum_{m=0}^{\infty}\left[\left(\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{2^n n!(n+1)!}\frac{\left(\bar{\alpha}_\mathrm{s}+a\bar{\alpha}_\mathrm{s}^2\right)^{n+1}}{(1/2+\mathrm{i}\nu+m-b\bar{\alpha}_\mathrm{s})^{2n+1}}\right)\right.\right.\\ &-\frac{\bar{\alpha}_\mathrm{s}}{1/2+\mathrm{i}\nu+m}-\bar{\alpha}_\mathrm{s}^2\left(\frac{a}{1/2+\mathrm{i}\nu+m}+\frac{b}{(1/2+\mathrm{i}\nu+m)^2}\right.\\ &\left.\left.\left.-\frac{1}{2(1/2+\mathrm{i}\nu+m)^3}\right)\right]\right\}\end{aligned} \tag{22}$$

is the solution of the ω-shift equation obtained in [78], with

$$a=\frac{5}{12}\frac{\beta_0}{N_c}-\frac{13}{36}\frac{n_f}{N_c^3}-\frac{55}{36}\,,\qquad b=-\frac{1}{8}\frac{\beta_0}{N_c}-\frac{n_f}{6N_c^3}-\frac{11}{12}. \tag{23}$$

The function $\chi^{(1)}(\nu)$ is conveniently represented in the form

$$\chi^{(1)}(\nu)=-\frac{\beta_0}{8N_c}\left(\chi^2(\nu)-\frac{10}{3}\chi(\nu)-\mathrm{i}\chi'(\nu)\right)+\bar{\chi}(\nu)\,, \tag{24}$$

where

$$\begin{aligned}\bar{\chi}(\nu) &= -\frac{1}{4}\left[\frac{\pi^2-4}{3}\chi(\nu)-6\zeta(3)-\chi''(\nu)-\frac{\pi^3}{\cosh(\pi\nu)}\right.\\ &+\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3+\left(1+\frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right)\\ &\left.+4\phi(\nu)\right]\,,\end{aligned} \tag{25}$$

$$\phi(\nu)=2\int_0^1\mathrm{d}x\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6}-\mathrm{Li}_2(x)\right]\,,$$

$$\mathrm{Li}_2(x)=-\int_0^x\mathrm{d}t\frac{\ln(1-t)}{t}\,. \tag{26}$$

Here and below $\chi'(\nu)=\mathrm{d}(\chi(\nu))/\mathrm{d}\nu$ and $\chi''(\nu)=\mathrm{d}^2(\chi(\nu))/\mathrm{d}\nu^2$.

The $|\nu\rangle$ representations for the impact factors are given by the following expressions:

$$\frac{C_1^{(0)}(\mathbf{q}^2)}{\mathbf{q}^2}=\int_{-\infty}^{+\infty}\mathrm{d}\nu' c_1(\nu')\langle\nu'|\mathbf{q}\rangle\,,$$

$$\frac{C_2^{(0)}(\mathbf{q}^2)}{\mathbf{q}^2}=\int_{-\infty}^{+\infty}\mathrm{d}\nu c_2(\nu)\langle\mathbf{q}|\nu\rangle\,, \tag{27}$$

$$c_1(\nu)=\int\mathrm{d}^2\mathbf{q}C_1^{(0)}(\mathbf{q}^2)\frac{\left(\mathbf{q}^2\right)^{\mathrm{i}\nu-\frac{3}{2}}}{\pi\sqrt{2}}\,,$$

$$c_2(\nu)=\int\mathrm{d}^2\mathbf{q}C_2^{(0)}(\mathbf{q}^2)\frac{\left(\mathbf{q}^2\right)^{-\mathrm{i}\nu-\frac{3}{2}}}{\pi\sqrt{2}}\,, \tag{28}$$

and by similar equations for $c_1^{(1)}(\nu)$ and $c_2^{(1)}(\nu)$ from the NLA corrections to the impact factors, $C_1^{(1)}(\mathbf{q}^2)$ and $C_2^{(1)}(\mathbf{q}^2)$.

Following [79], we obtain the amplitude as a spectral decomposition on the basis of eigenfunctions of the LLA BFKL kernel:

$$\begin{aligned}\frac{\mathrm{Im}_s(\mathcal{A})}{D_1D_2} &= \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty}\mathrm{d}\nu\left(\frac{s}{s_0}\right)^{\bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\chi(\nu)}\\ &\times\alpha_\mathrm{s}^2(\mu_\mathrm{R})c_1(\nu)c_2(\nu)\left\{1+\bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)\right.\\ &+\bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R})\ln\left(\frac{s}{s_0}\right)\left[\bar{\chi}(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\right.\\ &\times\left.\left(-\chi(\nu)+\frac{10}{3}+\mathrm{i}\frac{\mathrm{d}\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{\mathrm{d}\nu}+2\ln(\mu_\mathrm{R}^2)\right)\right]\\ &\left.+\ln\left(\frac{s}{s_0}\right)\chi_\mathrm{RG}(\nu)\right\}.\end{aligned} \tag{29}$$

We find that

$$c_{1,2}(\nu)=\frac{\left(Q_{1,2}^2\right)^{\pm\mathrm{i}\nu-\frac{1}{2}}}{\sqrt{2}}\frac{\Gamma^2[\frac{3}{2}\pm\mathrm{i}\nu]}{\Gamma[3\pm2\mathrm{i}\nu]}\frac{6\pi}{\cosh(\pi\nu)}\,, \tag{30}$$

$$c_1(\nu)c_2(\nu)=\frac{1}{Q_1Q_2}\left(\frac{Q_1^2}{Q_2^2}\right)^{\mathrm{i}\nu}\frac{9\pi^3(1+4\nu^2)\sinh(\pi\nu)}{32\nu(1+\nu^2)\cosh^3(\pi\nu)}\,, \tag{31}$$

$$\begin{aligned}\mathrm{i}\frac{\mathrm{d}\ln(\frac{c_1(\nu)}{c_2(\nu)})}{\mathrm{d}\nu} &= 2\left[\psi(3+2\mathrm{i}\nu)+\psi(3-2\mathrm{i}\nu)\right.\\ &\left.-\psi\left(\frac{3}{2}+\mathrm{i}\nu\right)-\psi\left(\frac{3}{2}-\mathrm{i}\nu\right)-\ln\left(Q_1Q_2\right)\right].\end{aligned} \tag{32}$$

It can be useful to separate from the NLA correction to the impact factor the terms containing the dependence on

s_0 and β_0,

$$C^{(1)}(\mathbf{q}^2) = \int_0^1 \mathrm{d}z \frac{\mathbf{q}^2}{\mathbf{q}^2 + z\bar{z}Q^2} \phi_\parallel(z) \left[\frac{1}{4} \ln\left(\frac{s_0}{Q^2}\right) \right. \times \ln\left(\frac{(\alpha+z\bar{z})^4}{\alpha^2 z^2 \bar{z}^2}\right) + \frac{\beta_0}{4N_c}\left(\ln\left(\frac{\mu_\mathrm{R}^2}{Q^2}\right) + \frac{5}{3} - \ln(\alpha)\right) + \cdots \left. \vphantom{\frac{1}{4}} \right] . \tag{33}$$

Accordingly, one can write

$$c_{1,2}^{(1)}(\nu) = \tilde{c}_{1,2}^{(1)}(\nu) + \bar{c}_{1,2}^{(1)}(\nu) , \tag{34}$$

where $\tilde{c}_{1,2}^{(1)}(\nu)$ are the contributions from the terms isolated in the previous equation and $\bar{c}_{1,2}^{(1)}(\nu)$ represent the rest. In [79] it was found that

$$\frac{\tilde{c}_1^{(1)}(\nu)}{c_1(\nu)} + \frac{\tilde{c}_2^{(1)}(\nu)}{c_2(\nu)} = \ln\left(\frac{s_0}{Q_1Q_2}\right)\chi(\nu) + \frac{\beta_0}{2N_c}\left[\ln\left(\frac{\mu_\mathrm{R}^2}{Q_1Q_2}\right) + \frac{5}{3} + \psi(3+2\mathrm{i}\nu) + \psi(3-2\mathrm{i}\nu) - \psi\left(\frac{3}{2}+\mathrm{i}\nu\right) - \psi\left(\frac{3}{2}-\mathrm{i}\nu\right)\right] . \tag{35}$$

One can construct infinitely many representations of the amplitude, all of them equivalent within NLA accuracy. A particular one, motivated in [82], is to exponentiate all the scale-invariant part of the NLA kernel, obtaining

$$\frac{\mathrm{Im}_s(\mathcal{A})}{D_1D_2} = \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty} \mathrm{d}\nu \times \left(\frac{s}{s_0}\right)^{\bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\chi(\nu) + \bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R})\left(\bar{\chi}(\nu) + \frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}\right]\right) + \chi_\mathrm{RG}(\nu)} \times \alpha_\mathrm{s}^2(\mu_\mathrm{R}) c_1(\nu) c_2(\nu) \times \left[1 + \bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)} + \frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right) + \bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R}) \ln\left(\frac{s}{s_0}\right) \frac{\beta_0}{8N_c}\chi(\nu) \times \left(\mathrm{i}\frac{\mathrm{d}\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{\mathrm{d}\nu} + 2\ln(\mu_\mathrm{R}^2)\right)\right] . \tag{36}$$

Another possible representation of the amplitude, in some sense closer to the original idea of the BFKL approach, is the "series" representation, which reads

$$\frac{Q_1Q_2}{D_1D_2}\frac{\mathrm{Im}_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_\mathrm{s}(\mu_\mathrm{R})^2 \times \left\{ b_0 + a_0 \ln\left(\frac{s}{s_0}\right) + \sum_{n=1}^{\infty} \bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})^n \left[a_n \ln\left(\frac{s}{s_0}\right)^{n+1} + b_n\left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0,\mu_\mathrm{R})\ln\left(\frac{s}{s_0}\right)^{n-1}\right)\right]\right\} , \tag{37}$$

where the coefficients

$$\frac{b_n}{Q_1Q_2} = \int_{-\infty}^{+\infty} \mathrm{d}\nu c_1(\nu)c_2(\nu)\frac{\chi^n(\nu)}{n!} , \tag{38}$$

are determined by the kernel and the impact factors in LLA and

$$\frac{a_n}{Q_1Q_2} = \int_{-\infty}^{+\infty} \mathrm{d}\nu c_1(\nu)c_2(\nu)\chi_\mathrm{RG}(\nu)\frac{\chi^n(\nu)}{n!} \tag{39}$$

arise from the collinear improvement. The coefficients

$$d_n = n\ln\left(\frac{s_0}{Q_1Q_2}\right) + \frac{\beta_0}{4N_c}\left((n+1)\frac{b_{n-1}}{b_n}\ln\left(\frac{\mu_\mathrm{R}^2}{Q_1Q_2}\right) - \frac{n(n-1)}{2} + \frac{Q_1Q_2}{b_n}\int_{-\infty}^{+\infty}\mathrm{d}\nu (n+1) f(\nu) c_1(\nu)c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!}\right) + \frac{Q_1Q_2}{b_n}\left(\int_{-\infty}^{+\infty}\mathrm{d}\nu c_1(\nu)c_2(\nu)\frac{\chi^{n-1}(\nu)}{(n-1)!} \times \left[\frac{\bar{c}_1^{(1)}(\nu)}{c_1(\nu)} + \frac{\bar{c}_2^{(1)}(\nu)}{c_2(\nu)} + (n-1)\frac{\bar{\chi}(\nu)}{\chi(\nu)}\right]\right) \tag{40}$$

are determined by the NLA corrections to the kernel and to the impact factors. Here, $\bar{c}_{1,2}^{(1)}(\nu)$ represent the contribution without the terms depending on s_0 and β_0, and

$$f(\nu) = \frac{5}{3} + \psi(3+2\mathrm{i}\nu) + \psi(3-2\mathrm{i}\nu) - \psi\left(\frac{3}{2}+\mathrm{i}\nu\right) - \psi\left(\frac{3}{2}-\mathrm{i}\nu\right) . \tag{41}$$

We stress that the terms in the series representation (37) with the a_n coefficients are beyond the NLA, since, as one can easily see from (22), χ_RG is $\mathcal{O}(\bar{\alpha}_\mathrm{s}^3)$.

3 Numerical results

In this section we present some numerical results for the dependence on s of the BFKL amplitude calculated for the process under study, using both the "exponentiated" and the "series" representations derived in the previous section. Following [79], we will adopt the principle of minimal sensitivity (PMS) [84, 85] requiring, for each value of s, the minimal sensitivity of the predictions to the change of both the renormalization and the energy scale, μ_R and s_0. In previous studies, where the unimproved kernel was used, the optimal choices for μ_R and s_0 turned out to be very far from the kinematical scales of the process. Our aim is to see if and to what extent the inclusion of a collinear improvement leads to more "natural" values for the optimal scales. This would demonstrate that the RG-generated terms reproduce the essential subleading dynamics, thus stabilizing the perturbative series. In the following analysis we use the two-loop running coupling corresponding to the value $\alpha_s(M_Z) = 0.12$.

3.1 Symmetric kinematics

We consider here the $Q_1 = Q_2 \equiv Q$ kinematics, i.e. the "pure" BFKL regime, with $Q^2 = 24\,\mathrm{GeV}^2$ and $n_f = 5$. We start with the "exponentiated" representation, given in (36) and set $\ln(s/s_0) = Y - Y_0$, where $Y = \ln(s/Q^2)$ and $Y_0 = \ln(s_0/Q^2)$. We have looked for the optimal value for the scales μ_R and Y_0. In practice, for each fixed value of Y we have determined the optimal choice of these parameters for which the amplitude is the least sensitive to their variation. We have found that the amplitude is always quite stable under variation of both scales and exhibits generally only one stationary point (local maximum). We choose as optimal values of the parameters those corresponding to this stationary point.

The optimal values turned out to be typically $\mu_R \simeq 3Q$ and $Y_0 \simeq 2$. In comparison with [79], where the optimal choice was typically $\mu_R \simeq 10Q$, we can see that there is a remarkable move towards "naturalness". The fact that the inclusion of the RG-terms affects the optimal choice of μ_R more strongly than of Y_0 is not surprising, since the added terms depend on μ_R and not on Y_0. In Fig. 2 we show the result for the (imaginary part of the) "improved" amplitude compared with the result obtained in [82]. The curves are in good agreement at the lower energies, the deviation increasing for large values of Y. This is consistent with having a larger asymptotic intercept when the collinear improvements are taken into account. We have to remember, however, that the applicability domain of the BFKL approach is determined by the condition $\bar{\alpha}_s(\mu_R)Y \sim 1$, which for our typical optimal value of μ_R and for $Q^2 = 24\,\mathrm{GeV}^2$ means $Y \sim 6$. Around this value the discrepancy is not so pronounced.

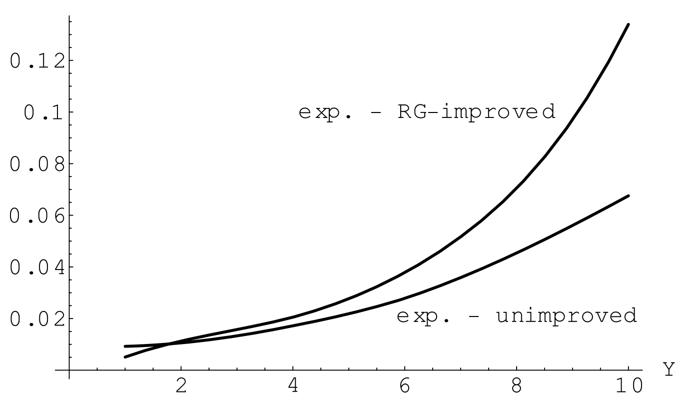

Fig. 2. $\mathrm{Im}_s(\mathcal{A})Q^2/(sD_1D_2)$ as a function of Y at $Q^2 = 24\,\mathrm{GeV}^2$ and $n_f = 5$ in the "exponentiated" representation with and without collinear improvement of the kernel; in both cases the PMS optimization method has been used

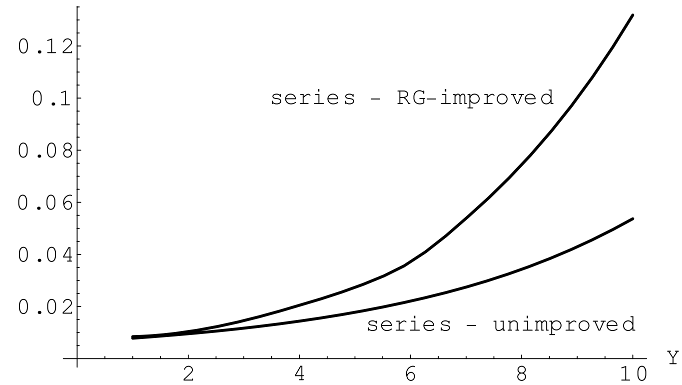

Fig. 3. $\mathrm{Im}_s(\mathcal{A})Q^2/(sD_1D_2)$ as a function of Y at $Q^2 = 24\,\mathrm{GeV}^2$ and $n_f = 5$ in the "series" representation with and without collinear improvement of the kernel; in both cases the PMS optimization method has been used

The next analysis has been done using the "series" representation of the amplitude, given in (37). In this case we have also observed a smooth dependence of the amplitude on the two energy parameters. The optimal values for Y_0 and μ_R turned out to be quite similar to those obtained for the "exponentiated" representation, $\mu_R \simeq 3Q$ and $Y_0 \simeq 3$. In Fig. 3 we show the behavior in Y of the "series" amplitude, compared with the determination of [79]. The situation is similar to Fig. 2, but the deviation between the curves appears to be more marked here. It is important to observe that the curves for the "exponentiated" and "series" representations of the amplitude as functions of Y with collinear improvement (see Figs. 2 and 3) fall almost on top of each other, while in the determination without the collinear improvement there was a discrepancy, more pronounced at higher energies [82]. This is a further indication of a better stability, induced by the collinear improvement.

In order to make visible the effect of the collinear improvement in the "series" representation we list the first few coefficients (see (37)) b_n, d_n, coming from the unimproved BFKL kernel and impact factors (in LLA e NLA respectively), and a_n, coming from the RG-resummed terms. Using the optimal scales chosen with the PMS method we obtain ($Q^2 = 24\,\mathrm{GeV}^2$, $n_f = 5$, $Y_0 = 3$, $\mu_R = 3Q$)

$$\begin{aligned} &b_0 = 17.0664\,, \quad b_1 = 34.5920\,, \quad b_2 = 40.7609\,,\\ &b_3 = 33.0618\,, \quad b_4 = 20.7467\,,\\ &d_1 = 0.674275\,, \quad d_2 = -1.73171\,,\\ &d_3 = -7.46518\,, \quad d_4 = -15.927\,,\\ &a_1 = 5.52728\,, \quad a_2 = 7.30295\,,\\ &a_3 = 6.42149\,, \quad a_4 = 4.24011\,. \end{aligned} \tag{42}$$

We can see that the a_n coefficients are of the opposite sigh with respect to the d_n, so "curing" the bad behavior of the BFKL series. Even if the values of the a_n coefficients decrease with n, they appear in (37) with two more powers of the energy logarithm than the d_n coefficients, so that their effect is not limited to low energies.

3.2 Asymmetric kinematics

When the virtualities of the photons are strongly ordered, we enter the "DGLAP" regime, where collinear effects should come heavily into the game. In this regime, previous attempts to numerically determine the amplitude using unimproved kernels were unsuccessful due to severe instabilities [83]. We have found here that these instabilities disappear if, instead, the RG-improved kernel is used.

In the numerical analysis to follow, we consider two choices for the virtualities of the photons, $Q_1 = 2\,\mathrm{GeV}$, $Q_2 = 12\,\mathrm{GeV}$ and $Q_1 = 0.5\,\mathrm{GeV}$, $Q_2 = 48\,\mathrm{GeV}$, so that $Q_1Q_2 = Q^2 = 24\,\mathrm{GeV}^2$ in both cases, and used the "exponentiated" representation. We define $Y = \ln(s/Q_1Q_2)$ and $Y_0 = \ln(s_0/Q_1Q_2)$.

For the first choice of virtualities, we find that for each Y value the amplitude is still quite stable under variation of the energy parameters and the optimal values are $\mu_R \simeq 4\sqrt{Q_1Q_2}$ and $Y_0 \simeq 2$, almost independently of Y. The same holds for the second choice of virtualities, with the only difference that now the optimal values depend strongly on Y. As an example, for $Y = 6$, when $\bar{\alpha}_s(\mu_R)Y \sim 1$, the optimal μ_R is $\simeq 3\sqrt{Q_1Q_2}$, but Y_0=7. This large value for Y_0 should not be surprising: if we use Q_2^2 as normalization scale in Y_0 instead of Q_1Q_2, the optimal value gets lowered down ~ 2.5, which looks more "natural".

In Fig. 4 we plot the amplitude for the two choices of photons' virtualities we have considered, together with the amplitude for $Q_1 = Q_2 = \sqrt{24}\,\mathrm{GeV}$. The amplitude becomes smaller and smaller when Q_2/Q_1 increases, as it must be expected due to the presence of the factor $\cos(\nu \log(Q_2^2/Q_1^2))$ in the integration over ν. We stress again that, if the RG-generated terms are removed, it is impossible even to draw the curves in Fig. 4 with $Q_2 \neq Q_1$.

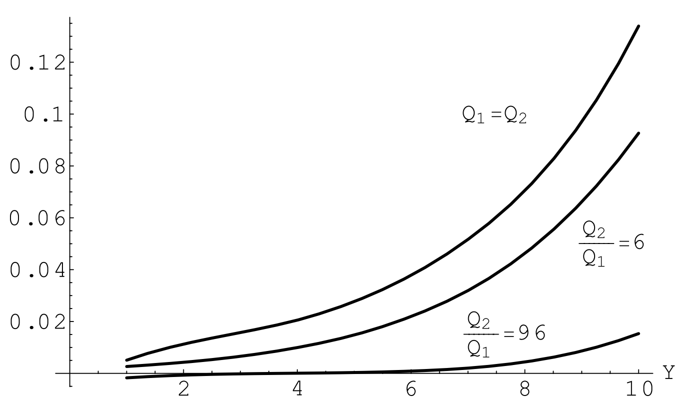

Fig. 4. $\mathrm{Im}_s(\mathcal{A})Q_1Q_2/(sD_1D_2)$ as a function of Y for photons with strongly ordered virtualities ($Q_2/Q_1 = 6$ and $Q_2/Q_1 = 96$, with $Q_1Q_2 = 24\,\mathrm{GeV}^2$), in comparison with the case of photons with equal virtualities ($Q_1^2 = Q_2^2 = 24\,\mathrm{GeV}^2$). All *curves* have been obtained using the "exponentiated" representation with the collinearly improved kernel

4 Conclusions

We have applied a RG-improved kernel to determine the amplitude for the forward transition from two virtual photons to two light vector mesons in the Regge limit of QCD with next-to-leading order accuracy. The result obtained is independent on the energy scale s_0, and on the renormalization scale μ_R within the next-to-leading approximation.

Using two different representations of the amplitude, which include the dependence on the energy scale and on the renormalization scale at subleading level, we have performed a numerical analysis both in the kinematics of equal and strongly ordered photons' virtualities.

An optimization procedure, based on the principle of minimal sensitivity, has led to results stable in the considered energy interval, which allow one to predict the energy behavior of the forward amplitude. The important finding is that the optimal choices of s_0 and μ_R are much closer to the kinematical scales of the problem than in previous determinations based on unimproved kernels. This effect is very marked for μ_R, as must be expected, since the extra terms depend on μ_R and not on s_0. This leads us to conclude that the extra terms in the BFKL kernel coming from collinear improvement, which are subleading to the NLA, catch an important fraction of the dynamics at higher orders.

Moreover, the use of the improved kernel has allowed us to obtain the energy behavior of the forward amplitude in the case of strongly ordered photons' virtualities, which turned out to be unaccessible to previous attempts using unimproved kernels.

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